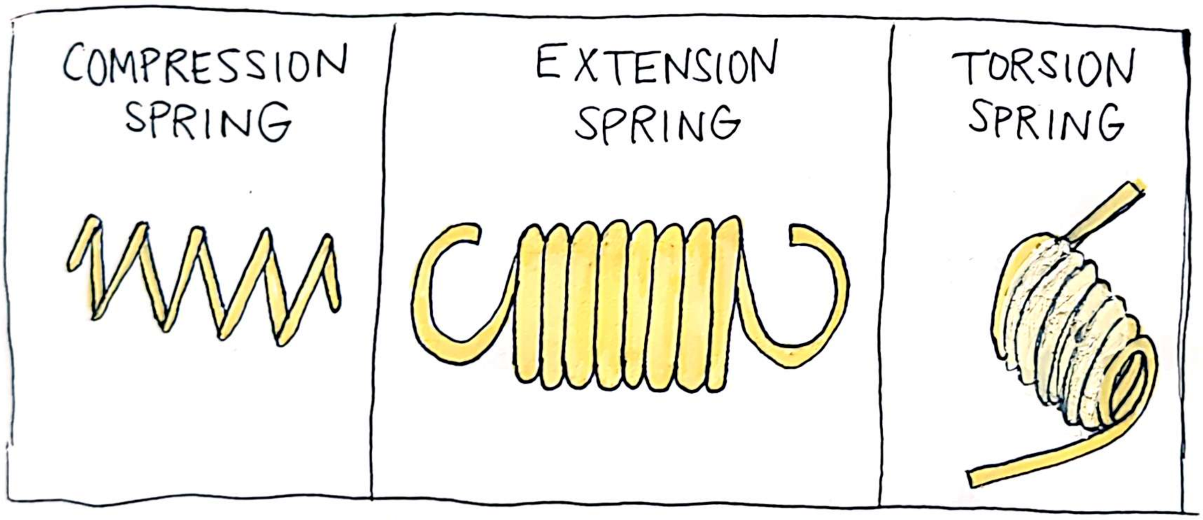
MECHANICAL SPRINGS

—— LECTURE NOTES —— MET 4501 —— PROF. LEAH GINSBERG ——

A SPRING IS A MACHINE COMPONENT THAT IS INTENTIONALLY SIGNIFICANTLY MORE COMPLIANT FROM THE OTHER PARTS IN A LOAD BEARING PATH.

SPRINGS ARE DESIGNED TO PROVIDE A PUSH, A PULL, OR A TWIST FORCE TO STORE/ABSORB ENERGY.



THE SPRING RATE k IS THE SLOPE OF THE LOAD-DEFLECTION CURVE.

FOR COMPRESSION/EXTENSION SPRINGS: $k = \frac{F}{y}$

FOR TORSION SPRINGS: $k = \frac{M}{\theta}$

SPRING RATES CAN BE CONSTANT (FOR LINEAR SPRINGS) OR VARY (FOR NONLINEAR SPRINGS)

SPRINGS CAN BE COMBINED IN SERIES OR PARALLEL.

- WHEN COMBINED IN SERIES, THE SAME FORCE PASSES THROUGH ALL SPRINGS, AND EACH SPRING CONTRIBUTES TO THE TOTAL DEFLECTION.

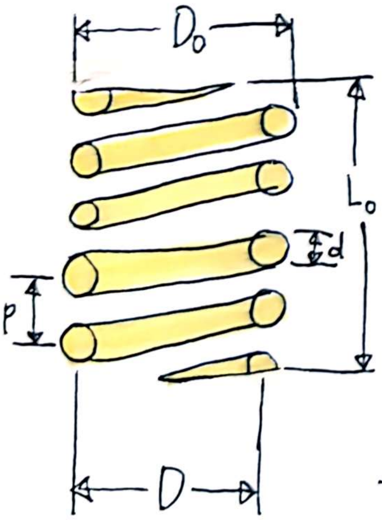
$$\frac{1}{k_{total}} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots + \frac{1}{k_n}$$

- WHEN COMBINED IN PARALLEL, ALL SPRINGS HAVE THE SAME DEFLECTION AND THE FORCE SPLITS AMONG THE INDIVIDUAL SPRINGS

$$k_{total} = k_1 + k_2 + \ldots + k_n$$

HELICAL COMPRESSION SPRINGS-GEOMETRY

LET'S LOOK AT A CROSS-SECTION OF A HELICAL SPRING.



THE WIRE DIAMETER IS d AND THE MEAN COIL DIAMETER IS D.

THE INNER DIAMETER (D_i) AND OUTER DIAMETER (D_o) CAN BE CALCULATED FROM D AND d.

$$D_i = D - d$$

$$D_o = D + d$$

THE SPRING INDEX (C) IS THE RATIO OF D AND d.

$$C = \frac{D}{d}$$

FOR MOST SPRINGS, $4 \leq C \leq 12$.

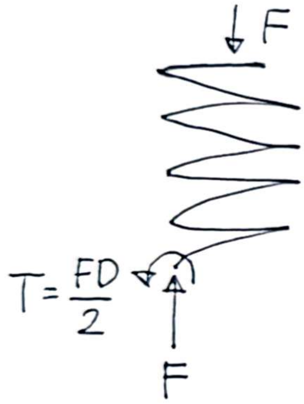
IF $C < 4$, DIFFICULT TO MANUFACTURE

IF $C > 12$, PRONE TO BUCKLING AND/OR TANGLING.

THE PITCH (p) IS THE AXIAL LENGTH BETWEEN ADJACENT COILS.

THE FREE LENGTH (L_0) IS THE OVERALL LENGTH OF THE SPRING IN THE UNLOADED CONDITION.

TO FIND THE STRESS IN THE SPRING, LETS TAKE A SECTION AND DRAW A FREE-BODY DIAGRAM.



THUS, THE MAXIMUM SHEAR STRESS IN THE WIRE IS:

$$\tau_{max} = \underbrace{\frac{Tr}{J}}_{\text{TORSIONAL SHEAR}} + \underbrace{\frac{F}{A}}_{\text{DIRECT SHEAR}}$$

MAKING SOME SUBSTITUTIONS AND ASSUMPTIONS, WE CAN ARRIVE AT:

$$\boxed{\tau = K_B \frac{8FD}{\pi d^3}}$$

WHERE K_B IS THE BERGSTRÄSSER FACTOR, $K_B = \frac{4C+2}{4C-3}$, WHICH IS A STRESS-CORRECTION FACTOR (SIMILAR TO A STRESS CONCENTRATION FACTOR).

THE SPRING RATE FOR A HELICAL SPRING CAN BE APPROXIMATED AS:

$$\boxed{k \approx \frac{d^4 G}{8D^3 N}}$$

(EQN 10-9 IN SHIGLEY)

WHERE G IS THE SHEAR ELASTIC MODULUS AND N IS THE NUMBER OF COILS (ACTIVE).

SEE TABLE 10-1 IN SHIGLEY FOR FORMULAS TO DETERMINE THE NUMBER OF ACTIVE COILS.

THE RECOMMENDED RANGE IS $3 \leq N_a \leq 15$

CLOSURE

WHEN ALL OF THE COILS OF THE SPRING ARE COMPRESSED TO THE POINT WHERE THEY TOUCH EACH OTHER, THE SPRING HAS "GONE SOLID."

THE HEIGHT OF A SPRING THAT HAS 'GONE SOLID IS CALLED THE SOLID HEIGHT, OR THE SOLID LENGTH (L_s).

F_s IS THE FORCE REQUIRED TO ACHIEVE CLOSURE ("GOING SOLID").

TO CHECK THAT THE OPERATING POINT OF THE SPRING IS SUFFICIENTLY FAR AWAY FROM GOING SOLID, WE DEFINE THE FRACTIONAL OVERRUN TO CLOSURE (ξ):

$$\boxed{F_s = (1+\xi)F_{max}}$$

WHERE F_{max} IS THE MAXIMUM FORCE APPLIED IN SERVICE.

FOR DESIGN PURPOSES, IT IS RECOMMENDED THAT $\xi \geq 0.15$

BUCKLING

COMPRESSION COIL SPRINGS MAY BUCKLE WHEN THE DEFLECTION BECOMES TOO LARGE. THE CRITICAL DEFLECTION IS DEFINED AS:

$$\boxed{y_{cr} = L_0 C_1' \left[1 - \left(1 - \frac{C_2'}{\lambda_{eff}^2}\right)^{1/2}\right]}$$

WHERE C_1' AND C_2' ARE DIMENSIONLESS ELASTIC CONSTANTS DEFINED AS:

$$C_1' = \frac{E}{2(E-G)} \quad ; \quad C_2' = \frac{2\pi^2(E-G)}{2G+E}$$

AND λ_{eff} IS THE EFFECTIVE SLENDERNESS RATIO, GIVEN BY THE EQUATION:

$$\lambda_{eff} = \frac{\alpha L_0}{D}$$

WHERE α IS THE END-CONDITION CONSTANT, WHICH DEPENDS ON HOW THE ENDS OF THE SPRING ARE SUPPORTED. VALUES FOR α ARE GIVEN IN TABLE 10-2.

IF A SPRING IS ABSOLUTELY STABLE, IT IS COMPLETELY RESISTANT TO BUCKLING. THE CONDITION FOR ABSOLUTE STABILITY IS

$$L_0 < \frac{\pi D}{\alpha}\left[\frac{2(E-G)}{2G+E}\right]^{1/2}$$

AS AN EXAMPLE, FOR A STEEL SPRING WITH SQUARED AND GROUND ENDS SUPPORTED BETWEEN FLAT PARALLEL SURFACES, THE CONDITION FOR ABSOLUTE STABILITY IS $L_0 < 5.26D$.

SPRING MATERIALS

THE IDEAL SPRING MATERIAL WOULD HAVE:

- HIGH ULTIMATE STRENGTH
- HIGH YIELD STRENGTH
- LOW MODULUS OF ELASTICITY

THE MOST COMMONLY USED SPRING MATERIALS ARE SUMMARIZED IN TABLE 10-3 OF SHIGLEY.

THE TENSILE STRENGTH OF SPRING MATERIALS DEPENDS ON THE WIRE SIZE. AS THE WIRE DIAMETER GETS SMALLER AND SMALLER, THE TENSILE STRENGTH APPROACHES THE STRENGTH OF ATOMIC BONDS.

$$\boxed{S_{ut} = \frac{A}{d^m}}$$

WHERE A AND m ARE CONSTANTS FOUND IN TABLE 10-4 OF SHIGLEY.

THE YIELD STRENGTH (S_y) AND TORSIONAL YIELD STRENGTH (S_{sy}) ARE FOUND IN TABLE 10-5 OF SHIGLEY.